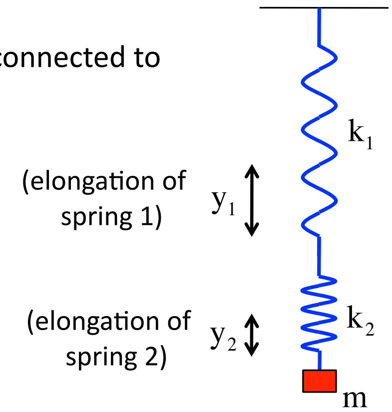
Problem 7.21

Two springs of different spring constant hang one connected to the other as shown.

a.) What is the total extension of the two-spring system when mass "m" is attached?



Each spring is supporting the same weight, "mg." That means:

$$F = -k_1 y_1$$

$$\Rightarrow \quad |y_1| = \frac{F}{k_1}$$

$$= \frac{mg}{k_1}$$

A similar relationship holds for the second spring, so the total displacement is:

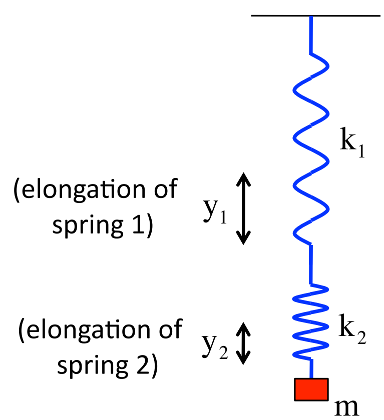
$$|y_{total}| = |y_1| + |y_2|$$

$$= \frac{mg}{k_1} + \frac{mg}{k_2}$$

$$= mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

b.) What's the effective spring constant?

If there was only one spring in the system, the displacement would look like:



$$|y_1| = \frac{mg}{k_1}$$

Putting the results from *Part a* into this form, we can write:

$$|y_{total}| = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = \frac{mg}{k_{effective}}$$

$$\Rightarrow \quad k_{effective} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

(Note that $k_{effective} < k_1$ and $k_{effective} < k_2$)

Conceptually, this makes sense. A single mass can elongate the two-spring system more than either of the springs making it up by themselves, so each individual spring in the system must be stiffer (that is what the spring constant measures) than the two springs put together. That is exactly what the math suggests (see "note" above).